Spin-dependent part of $p\bar{p}$ interaction cross section and Nijmegen potential.

V. F. Dmitriev,* A. I. Milstein,† and S. G. Salnikov‡

Budker Institute of Nuclear Physics, 630090 Novosibirsk, Russia and

Novosibirsk State University, 630090 Novosibirsk, Russia

(Dated: April 28, 2010)

Abstract

Low energy $p\bar{p}$ interaction is considered taking into account the polarization of both particles. The corresponding cross sections are calculated using the Nijmegen nucleon-antinucleon optical potential. Then they are applied to the analysis of the polarization buildup which is due to the interaction of stored antiprotons with polarized protons of a hydrogen target. It is shown that, at realistic parameters of a storage ring and a target, the filtering mechanism may provide a noticeable polarization in a time comparable with the beam lifetime.

*Electronic address: V.F.Dmitriev@inp.nsk.su

†Electronic address: A.I.Milstein@inp.nsk.su

‡Electronic address: salsergey@gmail.com

I. INTRODUCTION

An extensive research program with polarized antiprotons has been proposed recently by the PAX Collaboration [1]. This program has initiated a discussion of various methods to polarize stored antiprotons. One of the methods is to use multiple scattering on a polarized hydrogen target. If all particles remain in the beam (scattering angle is smaller than acceptance angle $\theta_{\rm acc}$), only spin flip can lead to polarization buildup, as was shown in Refs. [2, 3]. However, spin-flip cross section is negligibly small in both cases of proton-antiproton [2] and electron-antiproton [4] scattering. Hence the most realistic method is spin filtering [5]. This method implements the dependence of scattering cross section on orientation of particles spins. Therefore number of antiprotons scattered out of the beam after interaction with a polarized target depends on their spins, which results in the polarization buildup. Interaction with atomic electrons can't provide noticeable polarization because in this case antiprotons will scatter only in small angles and all antiprotons remain in the beam [2]. Thus it is necessary to study proton-antiproton scattering.

At present, quantum chromodynamics can't give reliable predictions for $p\overline{p}$ cross section below 1 GeV and different phenomenological models are usually used for numerical estimations. As a result, the cross sections obtained are model-dependent. All models are based on fitting of experimental data for scattering of unpolarized particles. These models give similar predictions for spin-independent part of the scattering cross sections, but predictions for spin-dependent parts may differ drastically.

Different nucleon-antinucleon potentials have similar behavior at large distance ($r \gtrsim 1\,\mathrm{fm}$) because long-range potentials are obtained by applying G-parity transformation to well-known nucleon-nucleon potential. The most important difference between nucleon-antinucleon and nucleon-nucleon scattering is existence of annihilation channels. A phenomenological description of annihilation is usually based on an optical potential of the form

$$V_{N\overline{N}} = U_{N\overline{N}} - iW_{N\overline{N}}. \tag{1}$$

Imaginary part of this potential describes annihilation into mesons and is important at small distance. The process of annihilation has no uniform description, and short-range potentials in various models are different.

Spin-dependent part of the cross section of $p\overline{p}$ interaction was previously calculated in Ref. [6] using the Paris potential. In Ref. [6] a possibility to obtain a noticeable beam polarization in a reasonable time was also investigated. Similar calculations were performed in Ref. [7] where various forms of Julich potentials were explored. Note that the contribution of interference between the Coulomb and strong amplitudes to the scattering cross section has been omitted in Ref. [7]. In the present paper, we calculate the spin-dependent part of the cross section of $p\overline{p}$ scattering using the Nijmegen model and analyze the polarization buildup which is due to the interaction of stored antiprotons with polarized protons.

II. CROSS SECTIONS

It is convenient to calculate the cross section in the center-of-mass frame, where antiproton and proton have momenta $\boldsymbol{p}$ and $-\boldsymbol{p}$, respectively. In the nonrelativistic approximation ($p \ll M$, where M is the nucleon mass), the antiproton momentum in the lab frame is $\boldsymbol{p}_{\rm lab} = 2\boldsymbol{p}$. Therefore the acceptance angle (maximum scattering angle when antiprotons remain in the beam) in the lab frame is connected with the acceptance angle in the center-of-mass frame by the relation $\theta_{\rm acc} = 2\theta^{(l)}_{\rm acc}$.

The $p\overline{p}$ scattering process has several channels: elastic scattering ($p\overline{p} \to p\overline{p}$), charge exchange ($p\overline{p} \to n\overline{n}$), and annihilation into mesons ($p\overline{p} \to mesons$). As explained above, noticeable polarization can be obtained only if some antiprotons are dropped out of the beam. The corresponding cross section can be written in the form

$$\sigma = \sigma_{\rm el} + \sigma_{\rm cex} + \sigma_{\rm ann}, \tag{2}$$

where $\sigma_{\rm cex}$ is the charge exchange cross section, $\sigma_{\rm ann}$ is the annihilation cross section, and $\sigma_{\rm el}$ is the elastic cross section integrated over scattering angle from $\theta_{\rm acc}$ to π. All cross sections are summed up over final spin states. The cross section $\sigma_{\rm el}$ includes pure Coulomb cross section, hadronic cross section, and interference term, which can't be omitted.

Spin-dependent cross section can be written in the form

$$\sigma = \sigma_0 + (\boldsymbol{\zeta}_1 \cdot \boldsymbol{\zeta}_2)\sigma_1 + (\boldsymbol{\zeta}_1 \cdot \boldsymbol{v})(\boldsymbol{\zeta}_2 \cdot \boldsymbol{v})(\sigma_2 - \sigma_1), \tag{3}$$

where $\boldsymbol{\zeta}_1$ and $\boldsymbol{\zeta}_2$ are unit vectors collinear to the particles spins, and $\boldsymbol{v} = \boldsymbol{p}/p$ is unit momentum vector. Here σ_0 is the spin-independent cross section, σ_1 describes spin effects

in the case when both vectors of polarization are perpendicular to $\boldsymbol{v}$, and σ_2 describes spin effects in the case when all three vectors are collinear. We direct quantization axis along the vector $\boldsymbol{v}$ and express the cross sections (3) via cross sections $\Sigma_{S\mu}$ calculated for states with total spin S and projection of total angular momentum μ:

$$
\begin{aligned}
\sigma_0 &= \frac{1}{2}\Sigma_{11} + \frac{1}{4}\left(\Sigma_{10} + \Sigma_{00}\right), \\
\sigma_1 &= \frac{1}{4}\left(\Sigma_{10} - \Sigma_{00}\right), \\
\sigma_2 &= \frac{1}{2}\Sigma_{11} - \frac{1}{4}\left(\Sigma_{10} + \Sigma_{00}\right).
\end{aligned}
\tag{4}
$$

Here we use the relation $\Sigma_{1-1} = \Sigma_{11}$.

The potential of proton-antiproton interaction is a sum of the Coulomb potential and optical Nijmegen potential [8]. Therefore the amplitude of elastic scattering can be written as a sum of the Coulomb amplitude and strong amplitude, which doesn't coincide with the amplitude calculated in the absence of the Coulomb field. Strong amplitude is not singular at small scattering angles, so that we can integrate hadronic cross section over the whole range from 0 to π. However, finite θ_{acc} should be taken into account at calculation of the Coulomb cross section and the interference between the Coulomb and strong amplitudes. In the nonrelativistic limit, the Coulomb amplitude is spin-independent and has the form

$$
F^{\mathrm{C}}_{1\mu} = F^{\mathrm{C}}_{00} = F^{\mathrm{C}}(\theta) = \frac{\alpha}{4vp\sin^2(\theta/2)} \exp\left\{-2i\eta \ln\left[\sin(\theta/2)\right] + 2i\chi_0\right\}, \tag{5}
$$

where $\chi_L = \arg\Gamma\left(L + 1 + i\eta\right)$ are the Coulomb phases, $\eta = -\frac{\alpha}{v_{\mathrm{lab}}}$ is the Sommerfeld parameter, $v_{\mathrm{lab}} = p_{\mathrm{lab}}/M$, and α is the fine structure constant.

For the strong elastic triplet scattering amplitude, we have

$$
\begin{aligned}
F^{\mathrm{el}}_{1\mu} &= \frac{i\sqrt{4\pi}}{2p} \sum_{m,L,J} C^{J\mu}_{Lm,1\mu-m} R^{J}_{L\mu} Y_{Lm}(\theta,\varphi), \\
R^{J}_{L\mu} &= \sum_{L'} (-1)^{\frac{L-L'}{2}} \sqrt{2L'+1}\, C^{J\mu}_{L'0,1\mu} \exp\left(i\chi_L + i\chi_{L'}\right)\left(\delta_{LL'} - S^{J}_{LL'}\right).
\end{aligned}
\tag{6}
$$

The sum over L, L' is performed under conditions $L, L' = J, J\pm 1$ and $|L - L'| = 0, 2$. Strong singlet amplitude reads

$$
F^{\mathrm{el}}_{00} = \frac{i\sqrt{4\pi}}{2p} \sum_{L} \sqrt{2L+1} \exp(2i\chi_L)\left(1 - S_L\right) Y_{L0}(\theta,\varphi). \tag{7}
$$

Here $S^J_{LL'}$ and S_L are partial elastic triplet and singlet scattering amplitudes, respectively, $Y_{Lm}(\theta,\varphi)$ are the spherical functions and $C^{J\mu}_{Lm,1\mu-m}$ are the Clebsch-Gordan coefficients.

Charge exchange scattering amplitudes have the form

$$
\begin{aligned}
F^{\mathrm{cex}}_{1\mu} &= -\frac{i\sqrt{4\pi}}{2p}\sum_{m,L,J} C^{J\mu}_{Lm,1\mu-m}\widetilde{R}^J_{L\mu}Y_{Lm}(\theta,\varphi),\\
\widetilde{R}^J_{L\mu} &= \sum_{L'}(-1)^{\frac{L-L'}{2}}\sqrt{2L'+1}C^{J\mu}_{L'0,1\mu}\exp\left(i\chi_{L'}\right)\widetilde{S}^J_{LL'}
\end{aligned}
\tag{8}
$$

and

$$
F^{\mathrm{cex}}_{00} = -\frac{i\sqrt{4\pi}}{2p}\sum_L \sqrt{2L+1}\exp(i\chi_L)\widetilde{S}_L Y_{L0}(\theta,\varphi). \tag{9}
$$

Here $\widetilde{S}^J_{LL'}$ and $\widetilde{S}_L$ are partial charge exchange triplet and singlet scattering amplitudes, respectively.

The cross sections $\Sigma_{1\mu}$ and Σ_{00} can be represented as a sum of pure Coulomb cross sections $\Sigma^{\mathrm{C}}_{1\mu}$ and Σ^{C}_{00}, hadronic contributions $\Sigma^{\mathrm{h}}_{1\mu}$ and Σ^{h}_{00}, and interference terms $\Sigma^{\mathrm{int}}_{1\mu}$ and $\Sigma^{\mathrm{int}}_{00}$. For the Coulomb contribution we have

$$
\Sigma^{\mathrm{C}}_{1\mu} = \Sigma^{\mathrm{C}}_{00} = \frac{\pi\alpha^2}{(vp\theta_{\mathrm{acc}})^2}, \tag{10}
$$

where smallness of θ_{acc} is taken into account. The total hadronic cross section can be calculated using the optical theorem

$$
\begin{aligned}
\Sigma^{\mathrm{h}}_{1\mu} &= \frac{2\pi}{p^2}\sum_{L,J}\sqrt{2L+1}C^{J\mu}_{L0,1\mu}\mathrm{Re}R^J_{L\mu},\\
\Sigma^{\mathrm{h}}_{00} &= \frac{2\pi}{p^2}\sum_{L}(2L+1)\,\mathrm{Re}\left[\exp(2i\chi_L)\,(1-S_L)\right].
\end{aligned}
\tag{11}
$$

The interference contributions can be calculated in the logarithmic approximation,

$$
\begin{aligned}
\Sigma^{\mathrm{int}}_{1\mu} &= -\frac{2\pi\alpha}{vp^2}\ln\left(\frac{2}{\theta_{\mathrm{acc}}}\right)\sum_{L,J}\sqrt{2L+1}C^{J\mu}_{L0,1\mu}\times\\
&\qquad\times\left\{\mathrm{Im}\left[\exp(-2i\chi_0)R^J_{L\mu}\right]+\frac{\alpha}{2v}\ln\left(\frac{2}{\theta_{\mathrm{acc}}}\right)\mathrm{Re}\left[\exp(-2i\chi_0)R^J_{L\mu}\right]\right\},\\
\Sigma^{\mathrm{int}}_{00} &= -\frac{2\pi\alpha}{vp^2}\ln\left(\frac{2}{\theta_{\mathrm{acc}}}\right)\sum_{L}(2L+1)\times\\
&\times\left\{\mathrm{Im}\left[\exp\left(2i\left(\chi_L-\chi_0\right)\right)(1-S_L)\right]+\frac{\alpha}{2v}\ln\left(\frac{2}{\theta_{\mathrm{acc}}}\right)\mathrm{Re}\left[\exp\left(2i\left(\chi_L-\chi_0\right)\right)(1-S_L)\right]\right\}.
\end{aligned}
\tag{12}
$$

The hadronic contributions to the elastic cross sections have the form

$$
\begin{aligned}
\Sigma^{\mathrm{el}}_{1\mu} &= \frac{\pi}{p^2}\sum_{L,J}\left|R^J_{L\mu}\right|^2 , \\
\Sigma^{\mathrm{el}}_{00} &= \frac{\pi}{p^2}\sum_{L}(2L+1)\left|1-S_L\right|^2 .
\end{aligned}
\tag{13}
$$

The charge exchange cross sections are given by

$$
\begin{aligned}
\Sigma^{\mathrm{cex}}_{1\mu} &= \frac{\pi}{p^2}\sum_{L,J}\left|\widetilde{R}^J_{L\mu}\right|^2 , \\
\Sigma^{\mathrm{cex}}_{00} &= \frac{\pi}{p^2}\sum_{L}(2L+1)\left|\widetilde{S}_L\right|^2 .
\end{aligned}
\tag{14}
$$

III. NUMERICAL RESULTS

We follow the method of calculations of scattering amplitudes S described in Refs. [8, 9]. To calculate S-matrix and cross sections (10 – 14), we use formula (15) of Ref. [8]. The partial cross sections obtained are in agreement with the values from Table V of Ref. [8].

Let us analyze the kinetics of polarization. Let $\boldsymbol{P}_T$ be the target polarization vector and $\boldsymbol{\zeta}_T = \boldsymbol{P}_T/P_T$. Antiproton beam polarization vector $\boldsymbol{P}_B$ is collinear to $\boldsymbol{\zeta}_T$ in both cases $\boldsymbol{\zeta}_T \perp \boldsymbol{v}$ and $\boldsymbol{\zeta}_T \parallel \boldsymbol{v}$. General solution of the kinetic equation is given in Refs. [2, 3]. In our case, when only spin-filtering mechanism is important, we have

$$
\begin{aligned}
P_B(t) &= \tanh\left[\frac{t}{2}\left(\Omega^{out}_{-}-\Omega^{out}_{+}\right)\right] , \\
N(t) &= \frac{1}{2}N(0)\left[\exp\left(-\Omega^{out}_{+}t\right)+\exp\left(-\Omega^{out}_{-}t\right)\right] ,
\end{aligned}
\tag{15}
$$

where

$$
\Omega^{out}_{\pm} = nf\left\{\sigma_0 \pm P_T\left[\sigma_1 + (\boldsymbol{\zeta}_T\cdot\boldsymbol{v})^2(\sigma_2-\sigma_1)\right]\right\} .
\tag{16}
$$

Here n is the areal density of the target and f is the beam revolving frequency. It follows from our calculations that $\left|\Omega^{out}_{-}-\Omega^{out}_{+}\right| \ll \left(\Omega^{out}_{-}+\Omega^{out}_{+}\right)$ which allows us to simplify Eq. (15). The beam lifetime due to the interaction with a target is

$$
\tau_b = \frac{2}{\Omega^{out}_{-}+\Omega^{out}_{+}} = \frac{1}{nf\sigma_0} .
\tag{17}
$$

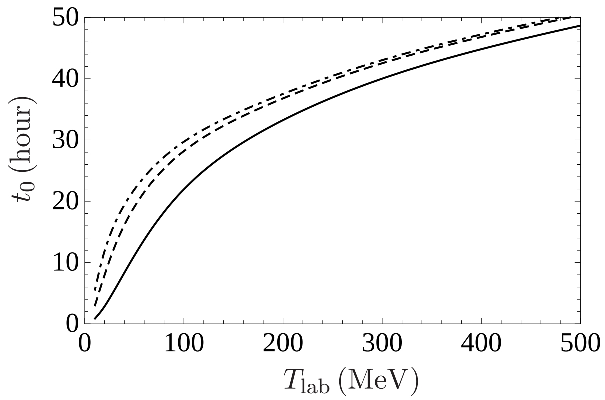

Figure 1: The dependence of t_0 (hour) on $T_{\rm lab}$ (MeV) for $n = 10^{14}\,\mathrm{cm}^{-2}$ and $f = 10^6\,\mathrm{c}^{-1}$. The acceptance angles in the lab frame are $\theta_{\rm acc}^{(l)} = 10\,\mathrm{mrad}$ (solid curve), $\theta_{\rm acc}^{(l)} = 20\,\mathrm{mrad}$ (dashed curve), $\theta_{\rm acc}^{(l)} = 30\,\mathrm{mrad}$ (dashed-dotted curve).

Note that Figure Of Merit $\mathrm{FOM}(t) = P_B^2(t)N(t)$ is maximal at $t_0 = 2\tau_b$, when the number of antiprotons is $N(t_0) \approx 0{,}14N(0)$. The polarization time t_0 as a function of the kinetic energy $T_{\rm lab}$ in the lab frame is shown in Fig. 1.

For the polarization degree at t_0, we have

$$P_B(t_0) = \begin{cases} -2P_T\dfrac{\sigma_1}{\sigma_0}, & \text{if } \boldsymbol{\zeta}_T \cdot \boldsymbol{v} = 0, \\ -2P_T\dfrac{\sigma_2}{\sigma_0}, & \text{if } |\boldsymbol{\zeta}_T \cdot \boldsymbol{v}| = 1. \end{cases} \tag{18}$$

Positive (negative) sign of $P_B(t_0)$ means that the beam polarization is parallel (antiparallel) to $\boldsymbol{\zeta}_T$.

The dependence of σ_1 and σ_2 on $T_{\rm lab}$ for different acceptance angles is shown in Fig. 2. These cross sections depend on the acceptance angle completely due the interference contribution. It was necessary to take interference into account in the case of pp scattering [2]. In the case of $p\overline{p}$ scattering the interference contribution is also important. Corresponding contributions $\sigma_{1,2}^{\rm int}$ are also shown in Fig. 2.

The dependence of polarization degree $P_B(t_0)$ for $P_T = 1$ on $T_{\rm lab}$ (MeV) for $\boldsymbol{\zeta}_T \cdot \boldsymbol{v} = 0$ ($P_\perp$) and $|\boldsymbol{\zeta}_T \cdot \boldsymbol{v}| = 1$ ($P_\parallel$) is shown in Fig. 3. In the case $\boldsymbol{\zeta}_T \cdot \boldsymbol{v} = 0$, the polarization degree becomes independent of antiproton energy at $T_{\rm lab}$ about $100\,\mathrm{MeV}$. With increasing acceptance angle the polarization degree raises faster. In the case $|\boldsymbol{\zeta}_T \cdot \boldsymbol{v}| = 1$, the polarization degree increases slower but amounts to 40% at energy about $200\,\mathrm{MeV}$.

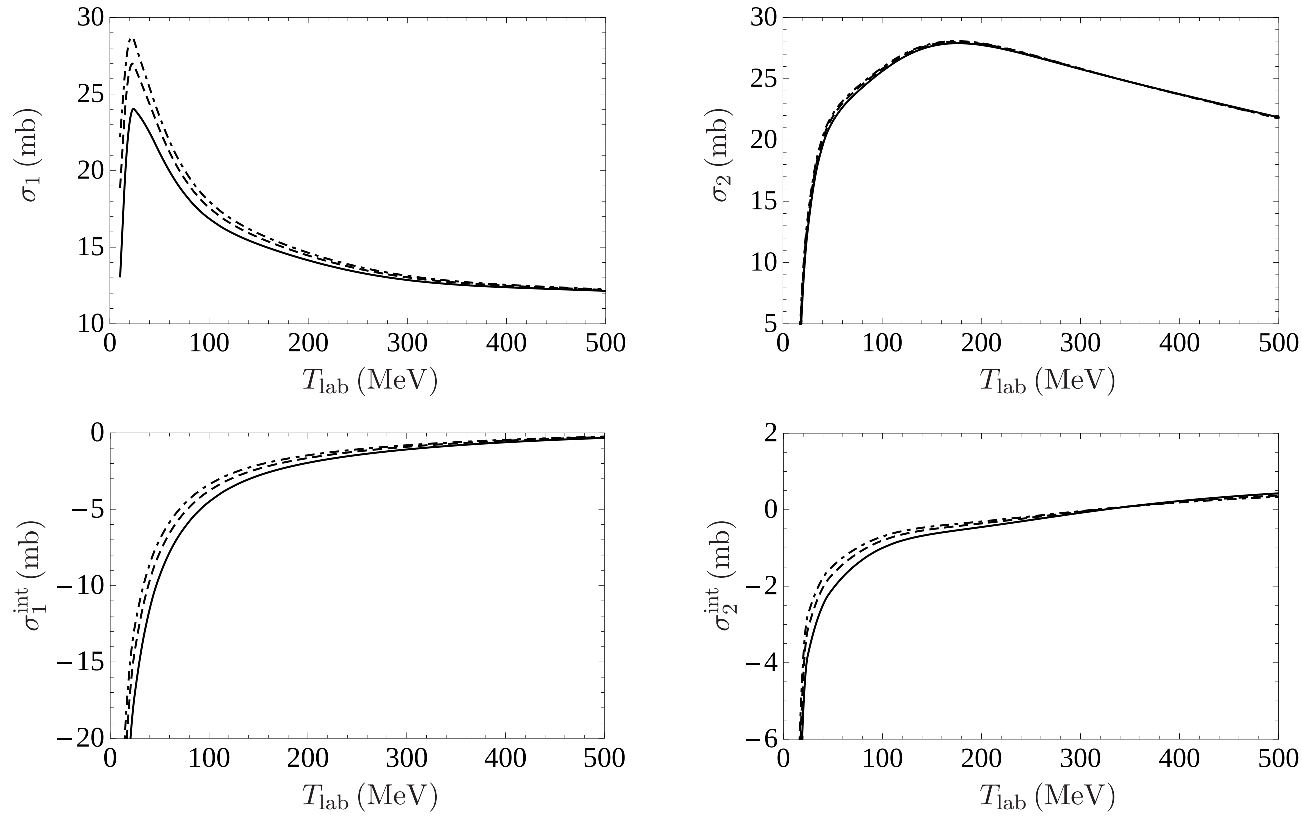

Figure 2: The dependence of σ_1, σ_2 and interference contributions σ_1^{int}, σ_2^{int} (mb) on T_{lab} (MeV). The acceptance angles in the lab frame are $\theta_{\mathrm{acc}}^{(l)} = 10\,\mathrm{mrad}$ (solid curve), $\theta_{\mathrm{acc}}^{(l)} = 20\,\mathrm{mrad}$ (dashed curve), $\theta_{\mathrm{acc}}^{(l)} = 30\,\mathrm{mrad}$ (dashed-dotted curve).

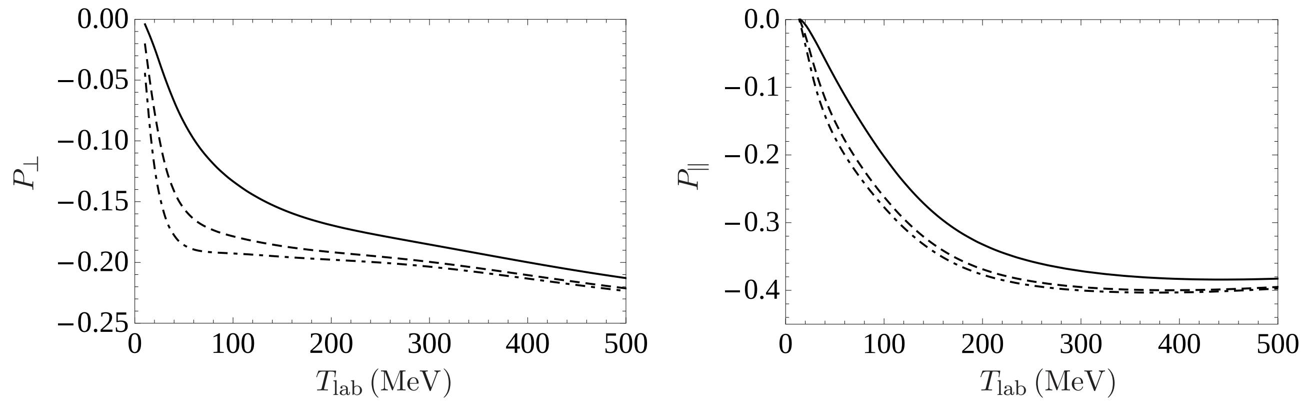

Figure 3: The dependence of $P_B(t_0)$ for $P_T = 1$ on T_{lab} (MeV) for $\boldsymbol{\zeta}_T \cdot \boldsymbol{v} = 0$ ($P_\perp$) and $|\boldsymbol{\zeta}_T \cdot \boldsymbol{v}| = 1$ ($P_\parallel$). The acceptance angles in the lab frame are $\theta_{\mathrm{acc}}^{(l)} = 10\,\mathrm{mrad}$ (solid curve), $\theta_{\mathrm{acc}}^{(l)} = 20\,\mathrm{mrad}$ (dashed curve), $\theta_{\mathrm{acc}}^{(l)} = 30\,\mathrm{mrad}$ (dashed-dotted curve).

Let us compare our results with the previous calculations. In Ref. [6], spin-dependent part of the cross section of $p\bar{p}$ scattering has been calculated using Paris potential in energy range $20 \div 100\,\mathrm{MeV}$. Authors predict positive $P_\perp$ with maximum 8% at energy about $60\,\mathrm{MeV}$ and negative $P_\parallel$, raising up to 12%. Analogous calculations have been performed in Ref. [7] using several modifications of the Julich model. Note that the contribution of interference between the Coulomb and strong amplitudes has been omitted. Qualitatively, the dependence of polarization degree on the antiproton energy, obtained in our paper, is similar to that in Ref. [7], but quantitative disagreement is rather big.

In conclusion, using the Nijmegen nucleon-antinucleon optical potential, we have calculated the spin-dependent part of the cross section of $p\bar{p}$ interaction and the corresponding degree of the beam polarization. Our results indicate that a filtering mechanism can provide a noticeable beam polarization in a reasonable time. However, we state that today it is impossible to predict the beam polarization with high accuracy because different models give essentially different predictions. Only experimental investigation of the spin-dependent part of the cross section of $p\bar{p}$ scattering can prove the applicability of potential models. Nevertheless, since polarization degree in all models are rather big, we can hope that filtering mechanism can be used to get polarized antiproton beam.

Acknowledgments

We are grateful to V. M. Strakhovenko for valuable discussions. The work was supported in part by RFBR under Grant No. 09-02-00024.

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